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A THEORETICAL INVESTIGATION INTO THE GEOPHYSICAL PROCESS OF BANK FORMATION

BY

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA

FALL, 1977

ABSTRACT

Suspended load is continuously deposited on the banks of alluvial rivers with active channels. An increase in suspended load without change in hydraulic conditions is thought to induce narrower channels. Stream-bed instability of laterally plane beds initiated by the effect of suspended load transport in the transverse direction, deepening the centre and building the sides, can explain the geomorphological phenomenon of bank formation.

A linear analysis of lateral instability of an erodible stream-bed has been carried out. For certain conditions a growth in the amplitude of bank-like perturbations confirms the existence of stream-bed instability in the lateral direction resulting in the formation of banks. The question as to why spontaneous bank formation is not observed in laboratory flumes is approached.

ACKNOWLEDGEMENTS

The author sincerely extends his acknowledgements to Dr. G. N. Parker for his invaluable suggestions and constructive criticisms throughout the research.

The author extends his gratitude to Professor J. Longworth, the Chairman of the Department of Civil Engineering, and Dr. N. Rajaratnam, Professor of Hydraulics Division, without whose help and constant encouragements, this research would not have been complete.

The author also appreciates the work done by F. M. Dick in typing this thesis.

TABLE OF CONTENTS

CHAPTER PAGE

1 INTRODUCTION. 1

1.1 General Statement. 1

1.2 Objective. 1

2 REVIEW OF LITERATURE. 3

2.1 Definitions. 3

2.2 Erosion and Sedimentation in Natural Streams. 4

2.2.1 Concept 4

2.2.2 Measuring Erosion and Sedimentation. 6

2.3 River Sediment; Mode of Transport. 7

2.3.1 Concept 7

2.3.2 Sediment Load Transport Theories. . 7

2.4 Theory of Suspended Load 7

2.4.1 The Turbulence Models 7

2.4.2 Diffusion-dispersion Model. 8

2.4.2.1 Concept. 8

2.4.2.2 Equations of Suspended Sediment in Shear Flow . . 11

2.4.2.3 Vertical Eddy Diffusivity. 12

2.4.2.4 Transverse Turbulent Eddy Diffusivity. 13

2.4.2.5 Vertical Distribution of Suspended Sediment 15

2.5 Hyperconcentration of Suspended Sediment . 17

  2.5.1 Concept 17

  2.5.2 Hyperconcentration and Turbulence . 17

  2.5.3 Hyperconcentration and Von Karman Constant. 18

2.6 Stability of Erodible Beds 18

  2.6.1 Concept 18

  2.6.2 Flow Regime and Bedforms. 19

  2.6.3 Model Analysis. 19

    2.6.3.1 Concept. 19

    2.6.3.2 Hydraulic Models 20

    2.6.3.3 Potential Flow Models. . . 24

    2.6.3.4 Rotational Models. 27

3 STABILITY ANALYSIS. 29

3.1 The Theoretical Model Stream 29

3.2 Formulation of the Mathematical Model. . . 30

  3.2.1 Mass-conservation Equation for Suspended Load. 30

  3.2.2 Formulation of the Boundary Conditions. 32

    3.2.2.1 Water Surface Boundary Equation 32

    3.2.2.2 Bed Boundary Equations . . 32

      3.2.2.2.1 Bed Boundary Condition for Suspended Sediment. 32

      3.2.2.2.2 Equation of Conservation of Bed Sediment. . 34

3.2.3 Secondary Currents and Lateral Momentum Transfer 34

3.2.4 The Perturbed Equations of Sediment Transport 35

3.2.5 Evaluation of the Unperturbed Concentration 37

3.2.6 Evaluation of Perburbed Form for ξ. . . 38

3.2.7 The Dimensionless Sediment Transport Equations 38

3.2.8 The Dispersion Equation 40

3.2.9 The Analytical Solution 43

3.2.10 Establishment of Instability. 45

4 SUMMARY AND CONCLUSION 46

REFERENCES 48

LIST OF FIGURES

Figure		Page
2.1	Bedforms	5
2.2	Vertical Distribution of Suspended Sand	9
2.3	Lateral Distribution of Suspended Sediment Concentration	10
2.4	Coefficient of Eddy-diffusivity in Vertical Direction	14
2.5	Definition Sketch for Exner's (1925) Models	21
2.6	Definition Sketch for Kennedy's (1963) Potential Flow Model	25
3.1	Definition Sketch for Mass-conservation Equation for suspended load	31
3.2	Simulated Stream Section	41
3.3	Stability Diagram	44

CHAPTER 1

INTRODUCTION

1.1 GENERAL STATEMENT

Knowledge of the instability of the fluid-bed interface has been extended in recent years, allowing for investigation and explanation of various aspects of the geomorphological behavior of natural streams. Interaction of flowing water and an erodible bed results in various bedforms. Many of the same methods used to analyze such bedforms as dunes can be used to explain the formation of banks, which are actually large scale lateral bed configurations. In fact, equilibrium banks can be formed in overly wide channels as a result of an erosion-deposition process, caused by a systematic perturbation of the gross lateral transport of sediment.

1.2 OBJECTIVE

The objective of this analysis is to investigate the process of bank formation. The formulation of a mathematical model for this purpose involves five essential steps:

1. The equations of sediment conservation describing the erosion-deposition process, subject to the limitations imposed by two boundary conditions, i.e. those at the water-surface and the bed, are formulated.

2. The equations are linearized for slight perturbations about steady uniform flow.
3. The equations are put in non-dimensional form which provides an appropriate format for the final results.
4. Sinusoidal perturbations characteristic of incipient bank formation are introduced and the associated dispersion equation is analyzed.
5. Stable and unstable regimes of initially laterally flat channels are established. The latter corresponds to incipient bank formation. The associated conclusions constitute the final stage of the analysis.

CHAPTER 2

REVIEW OF LITERATURE

2.1 DEFINITIONS

Some of the terms frequently used in this report are defined below.

Instability

If the equilibrium state of a system, when perturbed slightly, exhibits a growth of initial perturbation, that equilibrium state of the system is said to be unstable.

Turbulent Diffusion

Turbulent diffusion is the process by which random motions satisfying the momentum and mass balance of fluids spread mass, momentum, etc. from regions of high concentration to regions of low concentration.

Bed Load and Suspended Load

Bed load is the relatively coarse part of the sediment load that moves along the stream bed. Suspended load is the relatively fine part of the sediment load that is distributed throughout the vertical section of a stream.

Erosion and Sedimentation

Erosion is the removal of soil particles from their environment. Erosion in streams refers to the removal of sediments by water. Sedimentation refers to the deposition of eroded soil particles.

Erosion and sedimentation, as referred to in this analysis, are confined to noncohesive material only. The reason for this is that the behavior of cohesive material is different from that of noncohesive materials: its complicated physiochemical properties would make the study intractable.

Dunes and Antidunes

The term dunes is commonly used for bed features whose variation in elevation is more or less out of phase with that of surface wave above, while antidunes are more or less in phase with surface elevation. Dunes and antidunes are shown in Figure 2.1.

Ripples

Individual dune patterns are referred to as ripples which are understood to be small bed forms, whereas dunes are large ones.

2.2 EROSION AND SEDIMENTATION IN NATURAL STREAMS

2.2.1 Concept

There are two important categories of channel erosion:

1. Stream-bed erosion
2. Stream-bank erosion

While bed erosion is mainly in the direction of flow, bank erosion is a lateral process, converted to a downstream process only as the eroded bank material is carried downstream or causes sedimentation on the stream-bed. Shear stress governs the erosion process in the longitudinal direction of flow. However, on a channel bank another force plays an equally

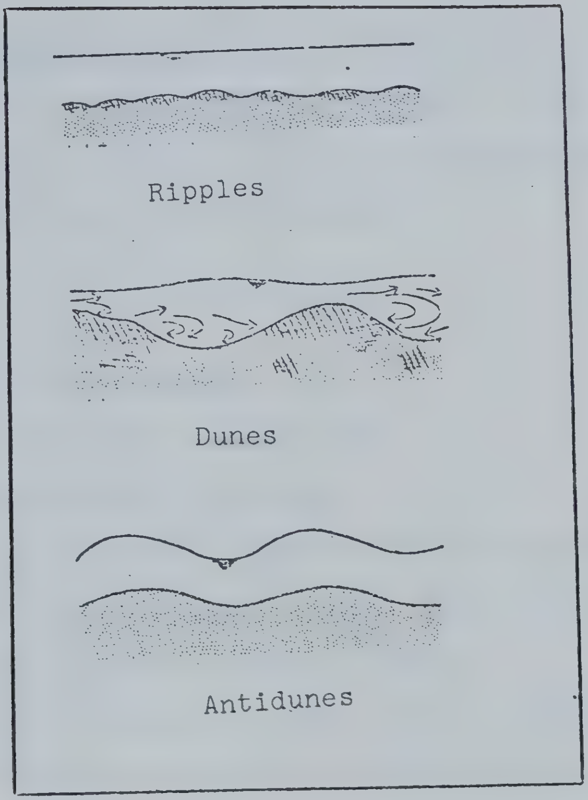

Figure 2.1 Bedforms

important role - the gravity force that causes particles to move down the sloping side of the channel. Lateral transport of suspended matter gives rise to sedimentation. It is the interaction between sedimentation and erosion that is assumed to form banks.

2.2.2 Measuring Erosion and Sedimentation

For channels in noncohesive sediments a tool for qualitative prediction of longitudinal erosive channel conditions is provided by Lane's (1955) relationship:

$$QS \quad \alpha \quad G_s \, d_s$$

where, Q = stream discharge
S = longitudinal stream slope
G_s = bed sediment discharge
d_s = a characteristic diameter of the bed material

This proportion is particularly useful when two of the four variables can be assumed to remain constant.

Quantitative estimates of gross channel erosion or sedimentation are obtained from time sequence comparisons of surveyed cross-sections (Vanoni, 1975). They can be indirectly estimated by relating sediment discharge to flow regime, a change in which can reflect the rate of erosion of sedimentation.

The above measurement techniques may be applied to a river that exhibits an unstable section due to excessive sedimentation or erosion; but for a river that is statistically stable, i.e. a river that is neither widening nor narrowing its channel on

an average basis, very little information is available to compute either the rate of erosion or deposition of bank material.

2.3 RIVER SEDIMENT; MODE OF TRANSPORT

2.3.1 Concept

In general, in a river, sediments ranging in size from clay particles to boulders may be found. Depending upon the mode of transportation, the sediments are classified as either bed load or suspended load. As far as size is concerned, there is no clear demarcation between suspended and bed load. A particular river under a particular flow situation may transport specified size ranges of suspended and bed load materials; but this range may not correspond to another river with a different flow condition.

2.3.2 Sediment Load Transport Theories

The mode of transport of bed load is different from that of suspended load; the former being transported in a saltating, or jumping mode while in close proximity to stream-bed; and the latter being transported in suspension while being distributed over the entire section of the stream. The precise mechanism of bed load transport is out of the scope of this study. The theory of suspended load transport is described separately in the following chapter.

2.4 THE THEORY OF SUSPENDED LOAD

2.4.1 The Turbulence Models

The suspension of sediment particles in a flow is attributed to turbulence. The turbulence model as formulated by Kennedy

(1895) states that the whole body of water is kept in circulation due to turbulence; this favours suspension.

Some of the earlier studies of turbulent suspension reveal three important facts that are also supported by experiment.

1. As regards to the vertical distribution of suspended matter, more suspended particles move near the bottom than near the water surface. This is illustrated in Figure 2.2, in which the suspended sand concentration distribution of four different sand diameters are plotted. The data are due to Kalinske and Pien (1943), collected at the centre of a 2.25 foot wide laboratory flume.
2. The lateral distribution of suspended matter is such that the suspended sediment concentration is higher near the centre than near the banks. This is illustrated in Figure 2.3, which is produced from the data of Hubbell and Matejka (1959), collected at three different times during one year period near Dunning, Nebraska.
3. A systematic variation of suspended sediment concentration in the downstream direction is probably not present over fairly short, uniform reaches with no tributaries.

2.4.2 Diffusion - Dispersion Models

2.4.2.1 Concept

The suspension of sediment particles due to turbulence is treated in the model as a diffusion-dispersion process. In this

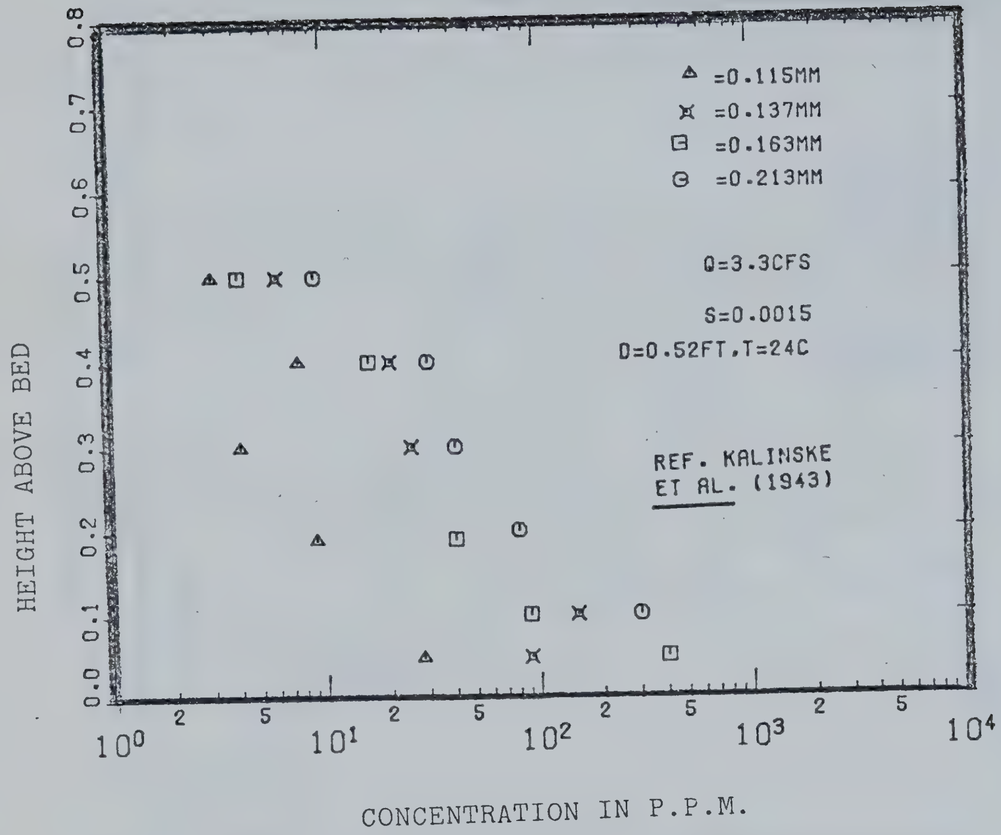

Figure 2.2 Vertical Distribution of Suspended Sand

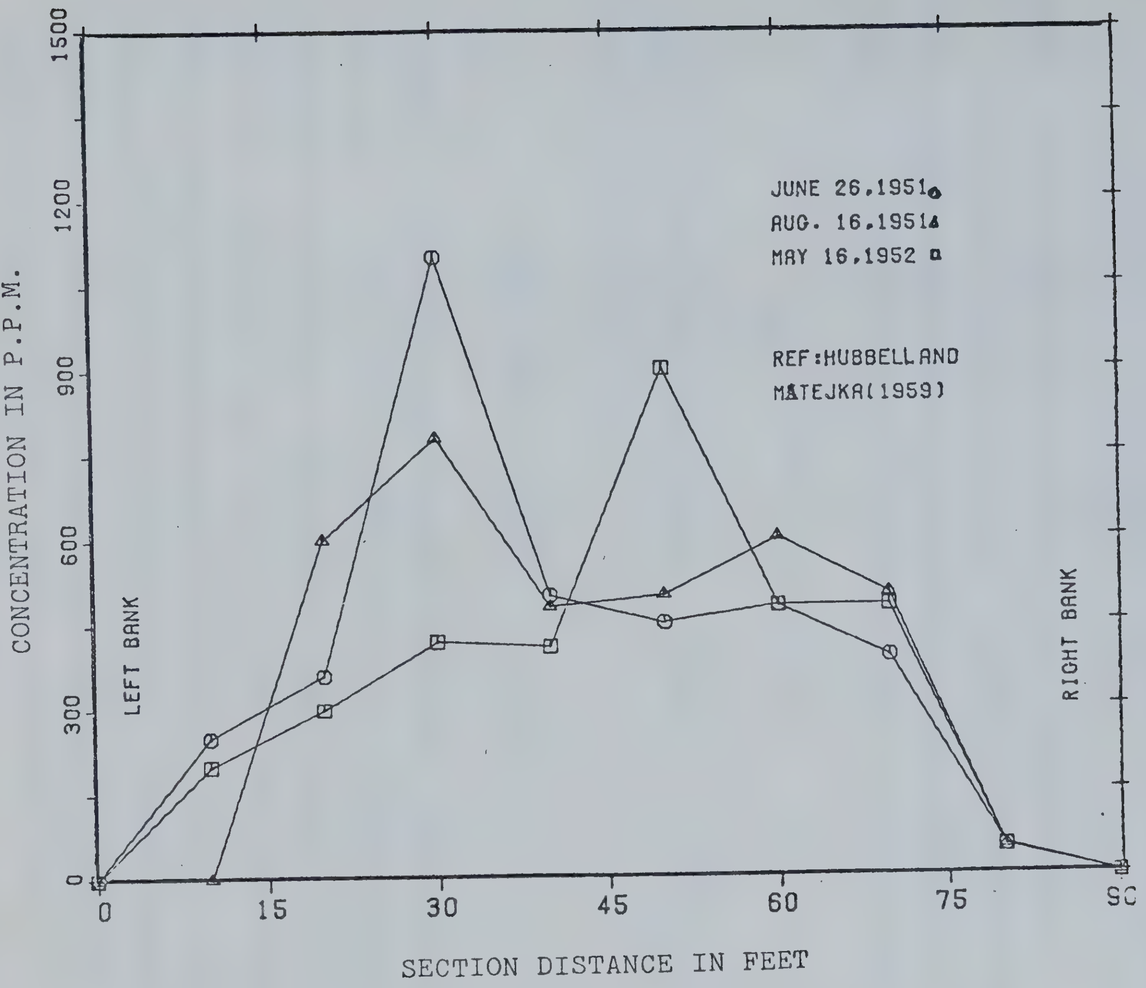

Figure 2.3 Lateral Distribution of Suspended Sediment Concentration

process, the particles are diffused in the medium while being transported.

2.4.2.2 Equations of Suspended Sediment in Shear Flow

In a laminar flow, diffusion means the spreading by random motion, which occurs whether the fluid is moving or not.

If c is the concentration of a conservative substance and μ_c is the molecular kinematic diffusivity, the equation of conservation of the substance in a motionless fluid is given by,

$$\frac{\partial c}{\partial t} = \mu_c \nabla^2 c \qquad (2.1)$$

Equation 2.1 describes the diffusion process. If the velocity vector of the laminar flow is $\vec{U}$,

$$\frac{\partial c}{\partial t} + \vec{U}\nabla c = \mu_c \nabla^2 c \qquad (2.2)$$

Equation 2.2 describes the dispersion process, which combines the action of convection and diffusion. In a turbulent flow, molecular diffusion is far weaker than the turbulent diffusion induced by convective motion of the eddies. Thus in an averaged form of (2.2) the scalar molecular diffusivity can be replaced with an effective eddy diffusivity tensor. If this tensor is assumed to be diagonal, the mean three-dimensional equation for the conservation of suspended material takes the form,

$$\frac{\partial c}{\partial t} + U_x\frac{\partial c}{\partial x} + U_y\frac{\partial c}{\partial y} + U_z\frac{\partial c}{\partial z} = \frac{\partial}{\partial x}(\xi_x\frac{\partial c}{\partial x}) + \frac{\partial}{\partial y}(\xi_y\frac{\partial c}{\partial y}) + \frac{\partial}{\partial z}(\xi_z\frac{\partial c}{\partial z}) \qquad (2.3)$$

where, c and (U_x, U_y, U_z) are now interpreted to be mean quantities.

Here ξ_x, ξ_y, and ξ_z are the turbulent kinematic eddy diffusivities in x, y, and z directions respectively. Also here x, y, z correspond to longitudinal, lateral, and vertical direction coordinates respectively.

2.4.2.3 Vertical Eddy Diffusivity

The vertical eddy diffusivity of fluid mass ξ_1 and the vertical eddy diffusivity of suspended mass ξ_2 are given by a general relationship,

$$\xi_2 = m\xi_1 \qquad (2.4)$$

where, m is a constant of proportionality. Graf (1971) cites from the literature that the value of m can be approximated as 1.

If ξ_m is the vertical turbulent mixing coefficient of momentum, the form due to Rouse (1937) can be written as,

$$\xi_m = \frac{z}{D}\left(1 - \frac{z}{D}\right) KDU_* \qquad (2.5)$$

where, K = Von Karman's constant

D = Depth of flow in z direction

U_* = Shear velocity at the bed

Experimental data from Kalinske *et al*. (1943) suggest that the turbulent transfer coefficients of mass and momentum are approximately equal, i.e.

$$\xi_m = \xi_1 = \xi_z$$

Hence, we may approximate that $\xi_2 = \xi_z$, so that ξ_z can be given by,

$$\xi_z = \frac{z}{D}\left(1 - \frac{z}{D}\right) KDU_* \qquad (2.6)$$

The distribution of ξ_z is seen to be parabolic in the vertical direction, as reported by Kalinske _et al_.. This is illustrated in Figure 2.4.

Fischer (1973) suggests that for practical purposes, the depth-averaged value of ξ_z,

$$\xi_z = .067\ DU_* \qquad (2.7)$$

may be used in place of (2.6).

Engelund (1970) suggests the value,

$$\xi_z = .077\ DU_* \qquad (2.8)$$

2.4.2.4 _Transverse Turbulent Eddy Diffusivity_

The nonisotropic nature of turbulence makes it difficult to establish an analytical relationship between vertical and transverse eddy diffusivities. Fischer (1973) provides a detailed discussion of empirical values of the transverse eddy diffusivity determined from experiments. These experiments are divided into three categories: (1) laboratory experiments with floating particles, (2) laboratory experiments with dissolved tracers, and (3) field experiments with dissolved tracers.

The laboratory floating particle experiments as mentioned above, yield an average surface dimensionless mixing coefficient

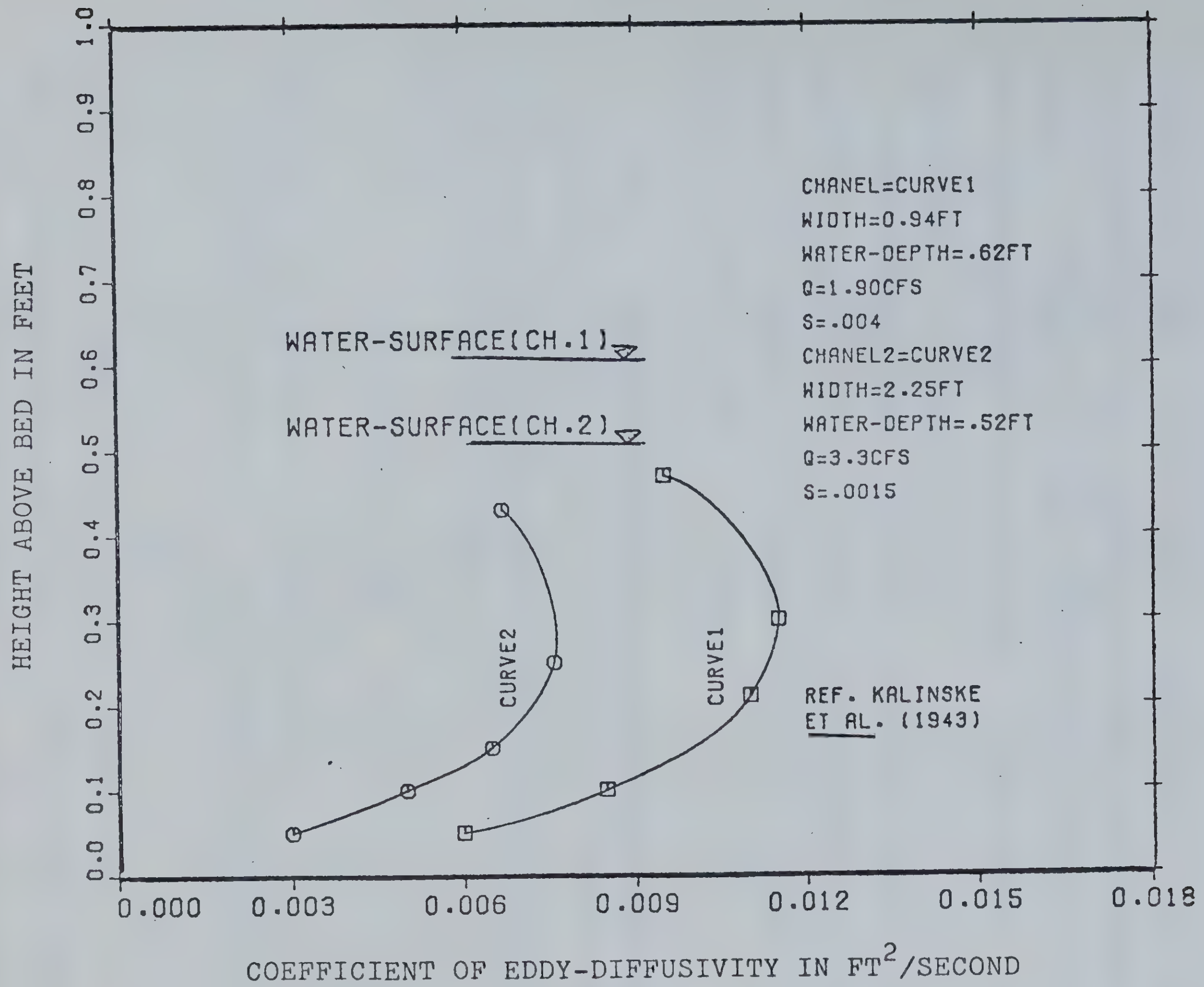

Figure 2.4 Coefficient of Eddy-Diffusivity in Vertical Direction

of $\xi_y/DU_* \approx 0.20$. An average of all laboratory experiments with dissolved tracers yields a depth-averaged transverse mixing coefficient $\xi_y/DU_* \approx 0.15$. In curving channels, the transverse mixing coefficient can be greatly accelerated by secondary currents. Fischer's findings indicate that in terms of DU_*, the transverse mixing coefficients for actual channels range from $\xi_y/DU_* = 0.51$ to 2.4.

Fisher quotes Okoye (1970) in his correlation of ξ_y/DU_* against the aspect ratio B/D, where B is the width of the channel. For values of B/D greater than 10, the depth-averaged value of ξ_y/DU_* is seen to be between 0.1 to 0.22.

2.4.2.5 Vertical Distribution of Suspended Sediments

Assuming a steady-state distribution of suspended matter that is uniform in the longitudinal and transverse directions, equation 2.3 can be integrated to yield,

$$0 = V_s c + \xi_z \frac{dc}{dz} \qquad (2.9)$$

where, V_s is the particle settling velocity.

Integrating equation 2.9, the solution is found to be,

$$c/Ca = e^{-\frac{V_s}{\xi_z}(z-a)} \qquad (2.10)$$

Here ξ_z is taken as a constant and Ca is a reference concentration at a height 'a' from the bed. If a vertically constant value of ξ_z is chosen, 'a' can be quoted to zero.

Equation 2.10 indicates that the concentration distribution is an exponential one, the concentration being larger closer to the bed than farther away from it. The steepness of the exponential curve depends upon the coefficient of diffusivity ξ_z and the settling velocity V_s. Comparison with field data, mainly for wide rivers, leads Lane and Kalinske (1941) to the conclusion that, equation 2.10, as approximate as it may be, is sufficiently accurate for practical purposes.

Engelund (1970) has found that with the use of equation 2.8, the nominal bed concentration Ca for a = 0 can be represented as,

$$Co = 0.0073 \left\{ \frac{U_*}{V_s} \right\}^3 \qquad (2.11)$$

This equation is accurate only up to $\frac{U_*}{V_s} \simeq 3$. For values of $\frac{U_*}{V_s}$ greater than 3, the volume concentration at the bed level becomes larger than 0.32. This value is suggested to be the maximum attainable volume concentration on the basis of experimental studies in flumes and field measurements of hyperconcentration (see section 2.5).

Another approach to suspended load distribution is due to Rouse (1964), who solves equation 2.9 with the form for ξ_z reported in the equation 2.5 to find,

$$c/_{Ca} = \left\{ \frac{D-z}{z} \cdot \frac{a}{D-a} \right\}^{Z+} \qquad (2.12)$$

where $Z+ = \frac{V_s}{DU_*}$, is the Rouse Number.

A feature of equation 2.13 is that z = 0 (bed level); the concentration becomes infinite; and hence the equation does not apply right at the bed where z = 0.

2.5 HYPERCONCENTRATION OF SUSPENDED SEDIMENT

2.5.1 Concept

Beverage et al. (1964) have defined hyperconcentrations to be those that exceed 40 percent by weight or 15 percent by volume.

The occurrence of extremely high, suspended sediment concentration is common to several alluvial streams. Beverage et al. have compiled some examples of hyperconcentrated reaches of such streams.

2.5.2 Hyperconcentration and Turbulence

The effect of high concentration on turbulence has been studied by Bagnold (1955), Nordin (1936) and Hino (1963). Many authors have discounted the role of turbulence as the major factor in suspending high concentrations of sediment. High damping of turbulence due to increases in suspended sediment concentration has been suggested by Bagnold. On the other hand, Hino predicts that the decrease in turbulence intensity due to hyperconcentration of suspended material should be very small. Hino's conclusion results from an analysis of the energy equation of flow with suspended particles, taking into account the major factors liable to influence flow with suspension. Beverage et al. quote from the literature that even at 68 percent

concentration by weight (28 percent by volume), streams have been observed to possess all the characteristics of fully-developed turbulence, with numerous intermittent sand waves and high velocities. In any case, the amount of damping realized in hyperconcentrations appears to be strongly dependent on particle size.

2.5.3. Hyperconcentration and the Von Karman Constant

In open channel flow without sediment, the Von Karman constant K, i.e. the universal constant in the logarithmic velocity law, is given approximately by K = 0.4. With an increase in suspended sediment concentration, K decreases. Graf (1971) quotes from the literature stating that a reduction in K is due to the damping effect of turbulence. Einstein *et al*. (1954) plot the rate of expenditure of frictional energy required to support the suspended particles versus K. The graph indicates that K decreases with increase in the rate of energy expenditure, which is directly proportional to the average concentration of suspended sediment. With the suspension of neutrally buoyant particles in water, Hino's investigation indicates that K decreases, and the turbulent intensity increases, with an increase in volume concentration of particles.

2.6 STABILITY OF ERODIBLE BEDS

2.6.1 Concept

The mathematical approach of Helmholtz (1888), quoted by Graf (1971) and Lamb (1945), indicates that a boundary between two fluids of different densities moving with different velocities

is subject to instability leading to interfacial waves. Graf quotes Baschin (1899) who suggested that the Helmholtz instability could explain bedforms (see chapter 2.6.2) if the loose sand can be considered to act like a fluid. Kennedy (1963) indicates that the instability of the sediment-fluid interface depends upon the depth and velocity of flow and the properties of sediment and fluid. The origin of meandering and braiding of alluvial rivers has been analysed using stability theory by Engelund and Hansen (1967), Callander (1969) and Parker (1976), to mention a few.

2.6.2 Flow Regime and Bedforms

In open channel flow the Froud number NF is often used as a flow criterion. For the purpose of classification of bedforms, three flow regimes may be distinguished: (1) a lower regime with $NF < 1$; (2) an upper regime with $NF > 1$; and (3) a transition between the upper and lower regimes with $NF \simeq 1$.

Idealized sketches of various bedforms are shown in Figure 2.1. The definitions of ripples, dunes, antidunes have been provided earlier in Section 2.1. Ripples and dunes usually occur in lower flow regime while antidunes are confined to upper flow regime. Washed-out dunes occur in the transition zone of the flow regime.

2.6.3. Model Analysis

2.6.3.1 Concept

Stability analysis of the stream-bed interface can provide theoretical predictions of the geometry and behavior of bedforms.

Different attempts to explain bedform mechanics have resulted in hydraulic, potential and rotational model analyses, some involving the stability analysis of the fluid-bed interface and some using the classical hydraulic approach. Reynolds (1975) provides a detailed description of each of the model studies.

2.6.3.2 Hydraulic Models

Graf (1971) quotes Exner's (1925) analytical treatment of an erodible stream-bed using a one dimensional flow approach. Exner's model is classical and is the predecessor of the hydraulic model. In this model, velocity variation with depth is reduced to a sectionally averaged mean velocity.

Referring to Figure 2.5, the governing equation in Exner's model is the equation of continuity, which may be written as,

$$(h - \eta)\, B(x)\, \overline{U} = Q = \text{constant} \qquad (2.13)$$

where, h is the watersurface height

η is the bed height from an arbitrary datum

$\overline{U}$ is the sectionally averaged velocity

$B(x)$ is the channel width that varies in the x direction of flow

Q is the flow rate

In progressive stages, the model takes into account effects due to bed friction which is related in a general fashion to the local depth and mean velocity, and the effect of nonlinear acceleration. The equation of motion is given by,

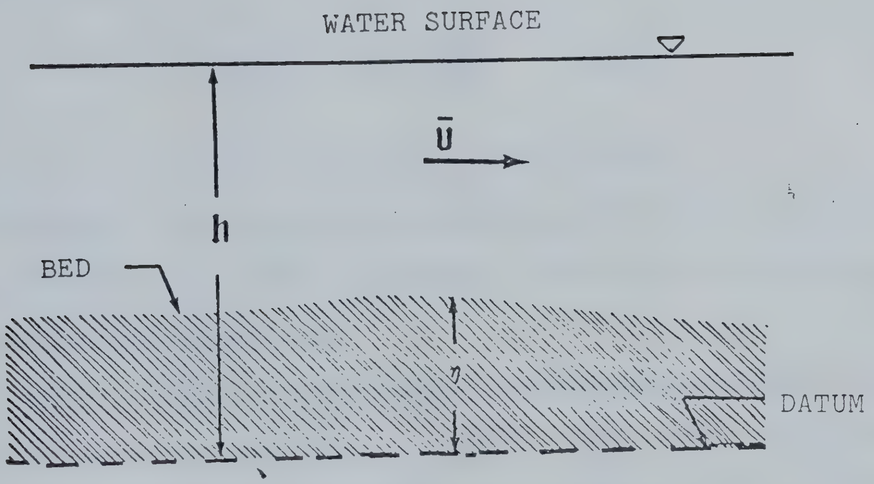

Figure 2.5 Definition Sketch for Exner's (1925) Model

$$\frac{\partial \overline{U}}{\partial t} = -\overline{U}\,\frac{\partial \overline{U}}{\partial x} - K\overline{U} + g_x - \frac{1}{p}\cdot\frac{\partial p}{\partial x} \qquad (2.14)$$

Here, the $K\overline{U}$ approximates frictional effects, g_x refers to the acceleration due to the body forces and $\frac{\partial p}{\partial x}$ is the pressure gradiant.

The change in bed height is given by Exner's erosion equation,

$$\frac{\partial n}{\partial t} = -\,a_E\,\frac{\partial \overline{U}}{\partial x} \qquad (2.15)$$

where, a_E is the Exner's erosion coefficient.

Equation 2.15 indicates that erosion occurs if the flow velocity increases (and deposition occurs if it decreases) in the downstream direction.

Exner's model is restricted to processes in which the flow depth is much smaller than the wavelength of the periodic bedwave. The model cannot predict the formation of bedforms, but provides a grossly correct picture of dune migration and the formation of steep lee fronts.

Gradowczyk's (1968) hydraulic model delineates wave propagation in one dimensional erodible bed channels using shallow-water wave theory for the fluid and a continuity equation for the bed sediment. Gradowczyk includes now-steady terms in the vertically integrated forms of the momentum and continuity equations, and indicates two 'dynamic' or surface dominated waves in addition to the 'kinematic' wave dominated by erosive

processes at the bed. These dynamic waves are in fact the usual shallow-water waves.

Gradowczyk's model predicts the steepening of the downstream faces of the dunes. The kinematic wave theory helps describe the non-linear development of bedforms, showing the interaction between faster-moving bedwaves and the slower-moving ones. Gradowczyk's hypothesis interprets the maximum growth conditions of bedwaves as that corresponding to the coincidence of crest and trough, as predicted by the kinematic wave theory.

Engelund and Hansen (1966) and Callander (1969) have generalized the vertically integrated model to include three dimensional flows. Here the local bed stress and transport rate is assumed to have the same direction as the velocity vector, which is uniform from the bed to the water surface. Nonsteady terms are included in the basic equations, as in the work of Gradowczyk but their contributions are neglected because they can be shown to be negligible for sediment-induced instability. The general nature of transition between dunes and antidunes is understood through the model of Engelund *et al*.. The detailed predictions of the mechanics of such bedforms is not possible through these models since the hydraulic model does not take suspended load into account. (The stability boundaries are critically dependent on the balance between the bed and suspended loads.)

2.6.3.3 Potential Flow Models

In studies based on potential flow models, the assumption of an irrotational flow of an ideal incompressible fluid subject to the influence of gravity is used.

Although the potential flow approach was applied to explain bedform mechanics by Anderson (1953), it was Kennedy (1963) who first used it in stability analysis.

The governing equation in Kennedy's potential flow model is,

$$\nabla^2 \phi = 0 \qquad (2.16)$$

where, ϕ is the potential function.

Referring to Figure 2.6, the boundary conditions at the free surface are given by,

$$U \frac{\partial \xi}{\partial x} + \frac{\partial \xi}{\partial t} = \frac{\partial \phi}{\partial z} \qquad \text{on } \underline{z} = 0 \qquad (2.17)$$

$$g\xi + U \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} = 0 \qquad \text{on } \underline{z} = 0 \qquad (2.18)$$

Equation 2.17 is the kinematic boundary condition and equation 2.18 expresses the dynamic condition of vanishing pressure at the free surface.

The boundary conditions at the fluid-bed interface are given by,

$$U \frac{\partial \eta}{\partial x} + \frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \qquad \text{on } \underline{z} = -d \qquad (2.19)$$

$$\frac{\partial G(x,t)}{\partial \eta} + B_* \frac{\partial \eta}{\partial t} = 0 \text{ on } \underline{z} = -d \qquad (2.20)$$

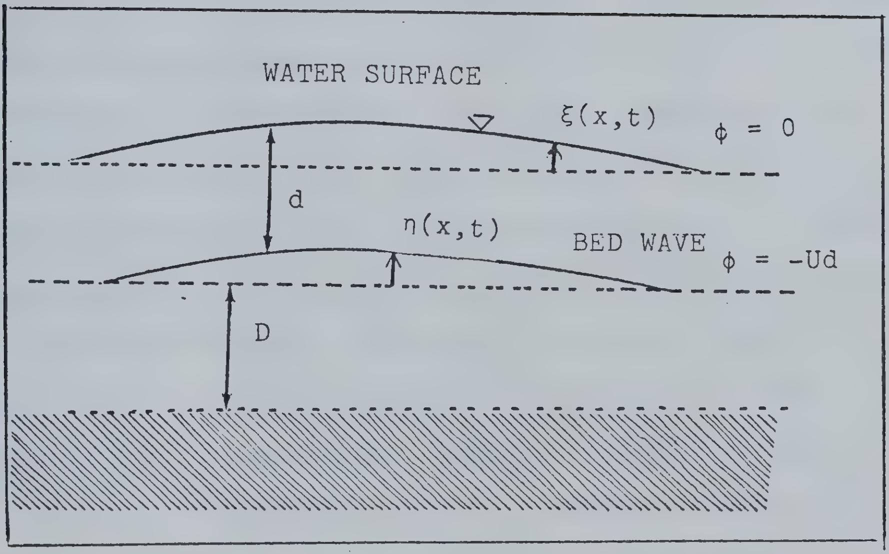

Figure 2.6 Definition Sketch for Kennedy's (1963) Potential Flow Model

Equation 2.19 expressed the kinematic condition at the bed and equation 2.20 is the sediment continuity equation, where $G(x,t)$ is the local rate of sediment transport per unit width by weight and B_* is the bulk specific weight of the sediment in the bed.

Kennedy's investigation concludes that the type of bedform and the wavelength of the bed features depend upon the Froude number, the depth of flow and an undetermined distance δ, by which the local sediment transport rate lags the local velocity. A modification to the interpretation of the lag distance δ was introduced by Hayashi (1970) through the addition of the assumption of dependance of the local transport rate on the bed slope as well as on the velocity. Recently Shirasuna (1973) propounds that the motion of the erodible bed is composed of the translation of a sandwave, whose travelling velocity is far smaller than the flow above it and the translation of a thin surface sediment layer which flows as a fluid. Based on those facts Shirasuna has formed an analytical model in which the motion of the fluid and the flowing surface layer at the bed are expressed as a two-layer potential flow of different densities. The sandwaves are treated as internal wave of small amplitude occurring at the interface between two potential layers. Although potential flow models outline the skeleton of the stability problem, they provide an incomplete view of the actual flow situations. The two severe limitations of these models are:

1. There is no account for bed shear stress.
2. The predicted velocity variation with depth is unrealistic.

2.6.3.4 Rotational Models

Rotational models are real fluid models which combine the essential features of hydraulic or vertically integrated models and potential models. The first significant contribution in this respect was made by Engelund (1970), who introduced most of the physical processes thought to be relevant to bed stability into an analytical framework.

The governing equation in Engelund's two dimensional rotational model is the vorticity transport equation,

$$\frac{d\omega}{dt} = \nu\nabla^2\omega \qquad (2.21)$$

where, ω is the vorticity of a two-dimensional flow

Both suspended and bed load are treated in the analysis.

With the single limitation of being confined to a two-dimensional flow, Engelund's model retains many advantages of the hydraulic models, accounting for friction and predicting bed shear. It also retains an essential feature of the potential flow model by accounting for the role of free surface for short waves. When suspended load was alone considered Engelund found that upper range instability (antidunes) was obtained, while the lower range remained stable. When the bed load was introduced, instability occured in the lower range as well, taking the form of dunes. With coarse sediments, the upper range instability disappeared, but the antidune range was little affected with fine sediments, for which the bed load is less important.

The results of the rotational model, no matter how successful they may be in predicting the mechanics of bedforms, apply mainly to sinusoidal long-crested bed waves. While these may be bedforms that develop first, they are not very like the asymmetric randomly placed, short-crested features usually found on streambeds.

CHAPTER 3

STABILITY ANALYSIS

3.1 THE THEORETICAL MODEL STREAM

The physical processes occurring in an actual river are extremely difficult to express in mathematical terms. A theoretical model river based on the following assumptions was conceived to facilitate the analysis of fluvial instability.

1. The model river is straight and has vertical frictionless banks.
2. The model river is free of bedforms.
3. The suspended sediment concentration and all other associated quantities stay constant along the longitudinal direction of the river.
4. The unperturbed suspended sediment concentration stays constant in the lateral direction.
5. Vertical eddy diffusivity is constant in the vertical direction, (Engelund 1969) and is given by,

$$\xi = .077\ U_* D$$

6. The diffusivity in the lateral direction is a constant multiple of that in the vertical direction.
7. Lateral movement of a bedload is neglected. That is, the equation of conservation of bed sediment does not

take into account that portion of the sediment load that moves in the lateral direction in a more or less close proximity to bed, i.e. the lateral bed load.

8. The velocity vector of suspended sediment is assumed to be equal to the fluid flow velocity vector plus the fall velocity vector.
9. In light of assumption 3 and 8, if the velocity vector of suspended sediment is assumed constant, the erosion rate at the bed may be assumed constant in the lateral direction.

3.2 FORMULATION OF THE MATHEMATICAL MODEL

3.2.1 Mass-Conservation Equation for Suspended Load

With reference to Figure 3.1, the mass-conservation equation for suspended sediment for an arbitrary control volume in the model river can be written as a flux balance equation in the form,

$$\frac{\partial c}{\partial t} + \frac{\partial Vc}{\partial y} + \frac{\partial}{\partial z}\left\{(W-V_s)c\right\} = \frac{\partial}{\partial z}\left\{\xi_z(y)\frac{\partial c}{\partial z}\right\} + \frac{\partial}{\partial y}\left\{\xi_y(y)\frac{\partial c}{\partial y}\right\} \qquad (3.1)$$

where, c is the suspended sediment concentration by volume

V_s is the fall velocity of the sediment

W and V are the mean fluid flow velocities in the upward (z) direction and lateral (y) direction respectively.

Herein, $\xi_y(y)$ and $\xi_z(y)$ are the eddy diffusivity coefficients in the lateral and vertical directions respectively.

From assumptions 5 and 6,

$$\xi_z(y) = \xi = .077\ U_*(y)D(y);$$

$$\text{and, } \xi_y(y) = B\xi_z(y) \qquad (3.2)$$

where B is a constant.

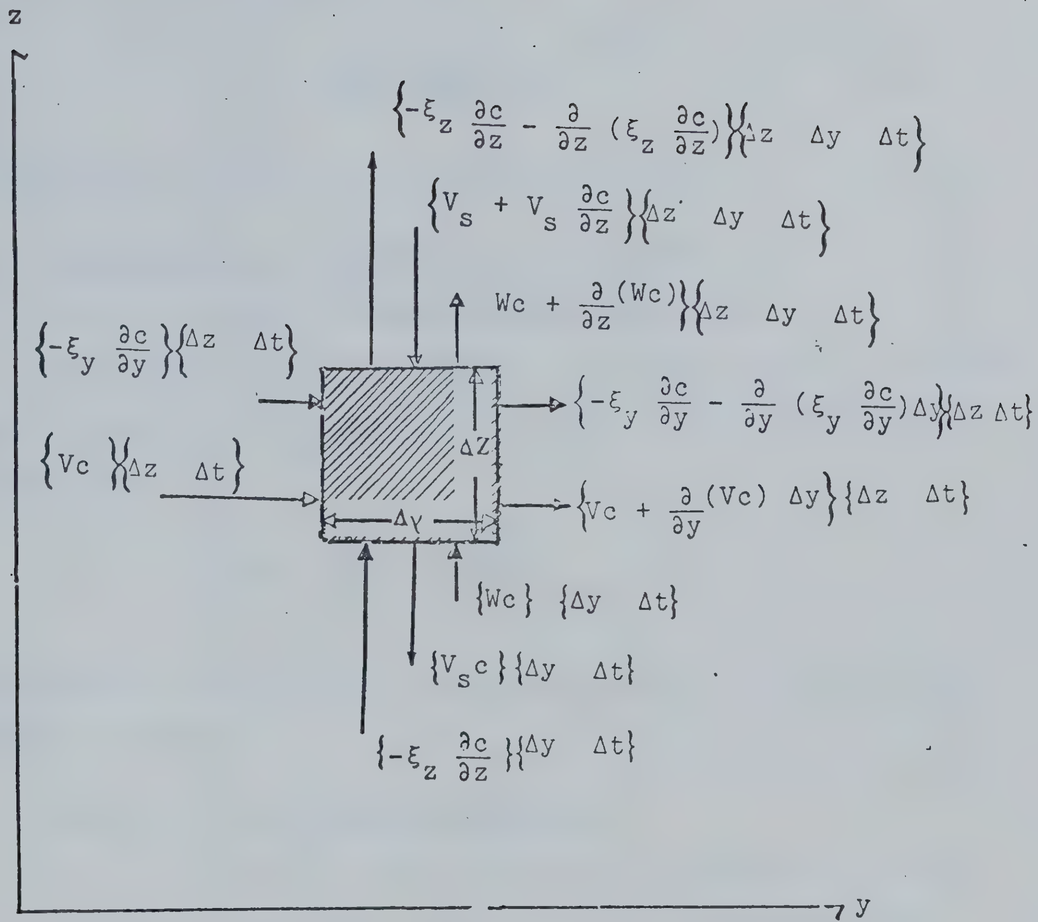
z
$\{-\xi_z \frac{\partial c}{\partial z} - \frac{\partial}{\partial z}(\xi_z \frac{\partial c}{\partial z})\}\{\Delta z \quad \Delta y \quad \Delta t\}$
$\{V_s + V_s \frac{\partial c}{\partial z}\}\{\Delta z \quad \Delta y \quad \Delta t\}$
$Wc + \frac{\partial}{\partial z}(Wc)\}\{\Delta z \quad \Delta y \quad \Delta t\}$
$\{-\xi_y \frac{\partial c}{\partial y}\}\{\Delta z \quad \Delta t\}$
$\{Vc\}\{\Delta z \quad \Delta t\}$
$\{-\xi_y \frac{\partial c}{\partial y} - \frac{\partial}{\partial y}(\xi_y \frac{\partial c}{\partial y})\Delta y\}\{\Delta z \ \Delta t\}$
ΔZ
$\{Vc + \frac{\partial}{\partial y}(Vc) \ \Delta y\}\{\Delta z \quad \Delta t\}$
ΔY
$\{Wc\} \ \{\Delta y \quad \Delta t\}$
$\{V_s c\}\{\Delta y \quad \Delta t\}$
$\{-\xi_z \frac{\partial c}{\partial z}\}\{\Delta y \quad \Delta t\}$
y

Figure 3.1 Definition Sketch for Mass-Conservation Equation for Suspended Load

Equation 3.1 can now be written in a modified form as:

$$\frac{\partial c}{\partial t} - (V_s - W)\frac{\partial c}{\partial z} = \xi\left[\frac{\partial^2 c}{\partial z^2} + B\frac{\partial^2 c}{\partial y^2}\right] + \frac{\partial c}{\partial y}\left[B\frac{\partial \xi}{\partial y} - V\right] \quad (3.3)$$

Here, V_s has been assumed to be constant and the fluid continuity equation for the mean flow,

$$\frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$$

has been used to simplify the convective terms.

3.2.2 Formulation of the Boundary Conditions

3.2.2.1 Water Surface Boundary Equation

Assuming that in the vertical (z) direction z = 0 at the water surface, the equation for the water surface boundary can be given as,

$$\left[-(V_s - W)c - \xi\frac{\partial c}{\partial z}\right]_{z=0} = 0 \quad (3.4)$$

The equation indicates that the vertical flux across the water surface is zero.

3.2.2.2 Bed Boundary Equations

The bed boundary is defined as the surface where z = -D in the vertical direction. At the bed two conditions apply:

1. Bed boundary equation for suspended sediment
2. Bed equation of mass conservation for bed sediment

3.2.2.2.1 Bed Boundary Condition for Suspended Sediment

If $\mathcal{D}$ is the temporal rate of deposition of suspended sediments on the bed from above per unit area and E_1 is the rate of

erosion from the bed per unit area due to vertical entrainment, then the net upward flux at the bed is given by the expression,

$$\left[-(V_s - W)c - \xi\frac{\partial c}{\partial z}\right]_{z = -D} = E_1 - \mathcal{D} \quad (3.5)$$

But $\mathcal{D} = (V_s - W)c\big|_{z = -D}$, since the downward convection flux due to fall velocity causes deposition of suspended sediment on bed.

Hence, equation 3.5 reduces to,

$$-\xi\frac{\partial c}{\partial z}\bigg|_{z = -D} = E_1 \quad (3.6)$$

If E is the previously defined vertical entrainment rate (given by $E = \frac{E_1}{V_s}$); the bed boundary equation for suspended sediment is given as,

$$-\xi\frac{\partial c}{\partial z}\bigg|_{z = -D} = V_s E \quad (3.7)$$

For a flat bed for unperturbed flow, let the concentration at the bed be called C_b. Since the rate of upward erosion must be equal to rate of downward deposition,

$$V_s C_b = V_s E \quad (3.8)$$

In equation 3.8 since the flow is unperturbed,

$$W = W_o = 0$$

Cancelling V_s from both the sides of the equation 3.8,

$$C_b = E \quad (3.9)$$

An appropriate evaluation for C_b can be made from equation 3.7.

3.2.2.2.2 Equation of Conservation of Bed Sediment

Conservation of bed sediment can be applied to provide the relation,

$$\left[-(V_s - W)c - \xi\frac{\partial c}{\partial z}\right]_{z = -D} = \frac{\partial D}{\partial t}(1 - p) \quad (3.10)$$

where, p = bed porosity

Typically p is equal to .3 ~ .4. It does not affect stability boundaries and so can be set equal to zero in a first analysis. Thus, neglecting porosity, equation (8.10) can be written as:

$$\left[-(V_s - W)c - \xi\frac{\partial c}{\partial z}\right]_{z = -D} = \frac{\partial D}{\partial t} \quad (3.11)$$

3.2.3 Secondary Currents and Lateral Momentum Transfer

Instabilities governed by sediment transport have characteristic time scales that are much larger than the characteristic time scales of the fluid flow itself. Thus it is valid to treat the fluid flow as time-invariant during the evolution of bed instability. This assumption is commonly referred to as quasi-steady assumption in the literature on the subject (e.g. Engelund, 1970). The cross-sectional velocities V and W are thus not generated by the instability itself. The only other mechanism for generating them is that which produces secondary flow. In an appropriately wide channel it may be surmised that these currents are negligible. To expedite the analysis, the following assumptions are made:

10. Secondary flows are neglected; $V = W = 0$

Equations 3.3, 3.4 and 3.11 become,

$$\frac{\partial c}{\partial t} - V_s \frac{\partial c}{\partial z} = \xi \left\{ \frac{\partial^2 c}{\partial z^2} + B \frac{\partial^2 c}{\partial y^2} \right\} + \frac{\partial c}{\partial y} \left\{ B \frac{\partial \xi}{\partial y} \right\} \qquad (3.12)$$

$$\left[-V_s c - \xi \frac{\partial c}{\partial z} \right]_{z = 0} = 0 \qquad (3.13)$$

$$\left[-V_s c - \xi \frac{\partial c}{\partial z} \right]_{z = -D} = \frac{\partial D}{\partial t} \qquad (3.14)$$

11. Secondary currents provide a mechanism for lateral redistribution of momentum. In the present analysis, we shall ignore all lateral momentum redistribution including that due to secondary currents,

12. Due to assumption 11, local bedstress in the downstream direction $\tau(y)$ can be calculated from the simple formula,

$$\tau = \rho g D(y) S \qquad (3.15)$$

where D(y) is local depth and S is the (constant) downstream water surface slope.

3.2.4 The Perturbed Equations of Sediment Transport

The mass balance equations are considered for slight perturbations about steady uniform flow conditions, i.e.

$$c = c_o(z) + c^1\ (y,z,t)$$

$$\xi = \xi_o + \xi^1$$

$$D = D_o(t) + D^1\ (y,t)$$

$E = E_o$, herein, E_o is the average entrainment rate, assumed be be constant in the lateral direction.

$$\tau = \tau_o + \tau^1$$

Introducing the above perturbations into the balance equations and reducing the equations read as follows,

Main equation:

$$-V_s\frac{dc_o}{dz} + \frac{\partial c^1}{\partial t} - V_s\frac{\partial c^1}{\partial z} = \xi_o \frac{d^2 c_o}{dz^2} + \xi^1 \frac{d^2 c_o}{dz^2} + \xi_o\left\{\frac{\partial^2 c^1}{\partial z^2} + B\frac{\partial c^1}{\partial Y^2}\right\} \quad (3.16)$$

Water surface boundary equation:

$$\left[-V_s c_o - \xi_o \frac{dc_o}{dz}\right]_{z = 0} + \left[-V_s c^1 - \xi_o \frac{\partial c^1}{\partial z} - \xi^1 \frac{\partial co}{\partial z}\right]_{z=0} = 0$$

(3.17)

Bed boundary equation for suspended sediment,

$$\left[-\xi_o \frac{dc_o}{dz} + - \xi_o\frac{\partial c^1}{\partial z} - \xi^1\frac{dc_o}{dz} + \xi_o D^1 \frac{d^2 c_o}{dz^2}\right]_{z=-D_o} = 0$$

(3.18)

Equation of conservation of bed sediment,

$$\left[-V_s c_o - \xi_o\frac{dc_o}{dz}\right]_{z = -D_o} + \left[-V_s c^1 + V_s D^1 \frac{dc_o}{dz} + \xi_o D^1\frac{dc_o}{dz^2} - \right.$$

$$\left. \xi_o\frac{\partial c^1}{\partial z} - \xi^1\frac{dc_o}{dz}\right]_{z = - D_o} = \frac{dD_o}{dt} + \frac{\partial D^1}{\partial t} \quad (3.19)$$

The zeroth order terms of these relations specify the unperturbed concentration, which must satisfy the equation,

$$-V_s\frac{dc_o}{dz} = \xi_o \frac{d^2 c_o}{dz^2} \quad (3.20)$$

and the boundary conditions,

$$\left[-V_s c_o - \xi_o \frac{dc_o}{dz}\right]_{z\,=\,0} = 0 \quad (3.21)$$

$$-\xi_o \frac{dc_o}{dz}\bigg|_{z\,=\,-D_o} = 0 \quad (3.22)$$

$$\left[-V_s c_o - \xi_o \frac{dc_o}{dz}\right]_{z\,=\,-D_o} = \frac{dD_o}{dt} \quad (3.23)$$

The perturbed concentration c^1 thus satisfies the equation,

$$\frac{\partial c^1}{\partial t} - V_s \frac{\partial c^1}{\partial z} = \xi_o \left\{ \frac{\partial^2 c^1}{\partial z^2} + B \frac{\partial^2 c^1}{\partial y^2} \right\} + \xi^1 \frac{d^2 c_o}{dz^2} \quad (3.24)$$

and the boundary conditions,

$$\left[-V_s c^1 - \xi_o \frac{\partial c^1}{\partial z} - \xi^1 \frac{dc_o}{dz}\right]_{z\,=\,0} = 0 \quad (3.25)$$

$$\left[-\xi_o \frac{\partial c^1}{\partial z} - \xi^1 \frac{dc_o}{dz} + \xi_o D^1 \frac{d^2 c_o}{dz^2}\right]_{z\,=\,-D_o} = 0 \quad (3.26)$$

$$\left[-V_s c^1 + V_s D^1 \frac{dc_o}{dz} + \xi_o D^1 \frac{d^2 c_o}{dz^2} - \xi_o \frac{\partial c^1}{\partial z} - \xi^1 \frac{dc_o}{dz}\right]_{z\,=\,-D_o} = \frac{\partial D^1}{\partial t} \quad (3.27)$$

Equation 3.25 is the water surface boundary equation, equation 3.26 is the bed boundary equation for suspended sediment and equation 3.27 is the equation of conservation of bed sediment.

3.2.5 Evaluation of the Unperturbed Concentration

The unperturbed concentration c_o can be evaluated from equation 3.20, which satisfies the equilibrium flow conditions.

Integrating equation 3.20 and evaluating the constants of integration from the two zeroth order boundary equations 3.21 and 3.22 results in the expression for c_o,

$$c_o = C_b \, e^{-\frac{V_s D_o}{\xi_o}\left(1 + \frac{z}{D_o}\right)} \qquad (3.28)$$

Herein, E has been substituted with Cb from the relation 3.9.

3.2.6 Evaluation of Perturbed Form for ξ

Referring to assumption 5,

$$\xi = .077\, U_* D, \text{ where } U_* = \sqrt{\tau/\rho}$$

From assumption 10, $\tau = \rho g D S$

Replacing U_* by τ in the expression for ξ,

$$\xi = .077 \sqrt{gDS} \; D \qquad (3.29)$$

In perturbed form equation 3.29 can be written as,

$$\xi = \xi_o + \xi^1 = .077 \sqrt{g(D_o+D^1)S} \; (D_o+D^1) \quad (3.30)$$

Rearranging the relation 3.30,

$$\xi_o + \xi^1 = \left\{.077 \sqrt{gD_oS} * D_o\right\}\left\{1 + \frac{3}{2}\frac{D^1}{D_o}\right\}$$

$$= \xi_o \left\{ 1 + \frac{3}{2}\frac{D^1}{D_o} \right\}$$

$$\text{Hence, } \xi^1 = \frac{3}{2} * \xi_o * \frac{D^1}{D_o} \qquad (3.31)$$

3.2.7 The Dimensionless Sediment Transport Equations

The relations for c_o and ξ^1 from equations 3.28 and 3.31 respectively are substituted into the equations 3.24, 3.25, 3.26, and 3.27. The resulting equations are expressed in dimensionless form using D_o as a length scale and V_s as a velocity scale. The resulting transformations are:

$$\frac{D^1}{D_o} = \tilde{D}$$

$$\frac{\xi_o}{D_o V_s} = \tilde{\xi}_o$$

$$\frac{z}{D_o} = \tilde{z}$$

$$\frac{y}{D_o} = \tilde{y}$$

$$\frac{t}{(D_o V_s)} = \tilde{t}$$

where, the terms with tildes on top indicate them in the dimensionless forms.

After dropping the tildes for sake of convenience, the mean equation for perturbed concentration can be written as follows,

$$\frac{\partial c^1}{\partial t} - \frac{\partial c^1}{\partial z} = \xi_o \left\{ \frac{\partial^2 c^1}{\partial z^2} + B \frac{\partial^2 c^1}{\partial y^2} \right\} + \frac{3}{2} \frac{C_b D}{\xi_o} e^{-(\frac{1 + z}{\xi_o})} \qquad (3.32)$$

Likewise, the water surface boundary equation is,

$$\left[-c^1 - \xi_o \frac{\partial c^1}{\partial z} + \frac{3}{2} C_b D e^{-(\frac{1}{\xi_o})} \right]_{z = 0} = 0 \qquad (3.33)$$

the bed boundary condition for suspended sediment is,

$$\left[-\xi_o \frac{\partial c^1}{\partial z} + \frac{C_b D}{\xi_o} + \frac{3}{2} C_b D \right]_{z = -1} = 0 \qquad (3.34)$$

and the equation of bed sediment conservation is,

$$\left[-c^1 - \xi_o \frac{\partial c^1}{\partial z} + \frac{3}{2} C_b D \right]_{z = -1} = \frac{\partial D}{\partial t} \qquad (3.35)$$

3.2.8 The Dispersion Equation

In hydrodynamic theory, a generally accepted procedure to analyse stability is by the assumption of sinusoidal forms for the perturbations. The balance equations then determine whether or not the amplitude of the perturbations increases or decreases. A growth in the amplitude indicates instability and a decay in the amplitude indicates stability. For neutral perturbations, the amplitude remains unchanged, corresponding to the transition between stable and unstable conditions.

In the present analysis, the entire lateral confinement of the bed is limited to one wavelength on which a single cosine wave has been imposed. The two crests of the wave are assumed to simulate the physical existence of banks. This is illustrated in Figure 3.2. A growth in the amplitude of the imposed waveform, as a result of erosion at the trough region and deposition at the crests, seems to correspond to the physical process of building banks. It is in this fashion that one is led to the viewpoint of bank formation as a stability process.

Forms for the perturbations in depth and concentration that mathematically express the above concept are,

$$c^1 = C_*(z) \cos ky \, e^{\alpha t} \qquad (3.36)$$

$$D = -D_*(z) \cos ky \, e^{\alpha t} \qquad (3.37)$$

Equation 3.36 introduces the idea of variation of perturbed suspended sediment concentration in a sinusoidal wave pattern

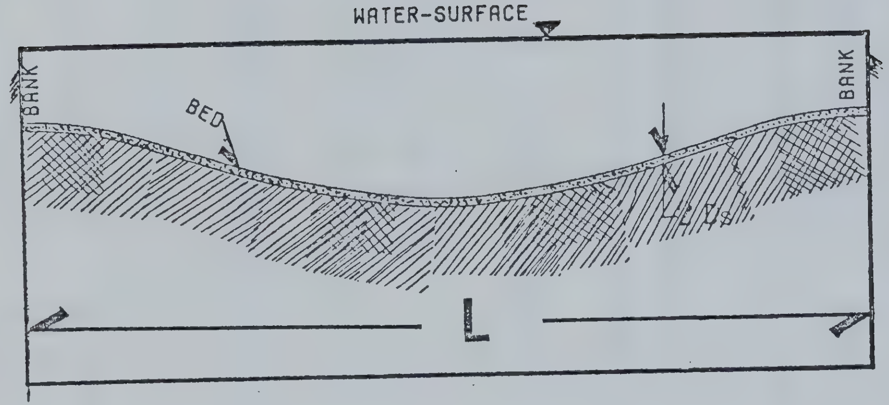

Figure 3.2 Simulated Stream Section

in the lateral direction. Equation 3.37 introduces the idea of sinusoidal bedwave in the lateral direction.

Here k is the dimensionless fluvial instability wave number, related to the dimensionless wavelength 'λ' by the relation $k = \frac{2\pi}{\lambda}$, where $\lambda = \frac{L}{D_o}$; L being the dimensional wavelength, equal to width of the stream. α here, is the time rate of change of amplitude (C_* in equation 3.36 and D_* in equation 3.37) of the wave.

The balance equations can be reduced with these forms to yield the following. For the main equations we have,

$$C''_*(z)\,\xi_o + C'_*(z) - C_*(z)\alpha - 3\xi_o C_*(z)k^2 = \frac{3}{2}\,\frac{C_bD}{\xi_o}\,e^{-\left(\frac{1+z}{\xi_o}\right)} \qquad (3.38)$$

The water surface boundary becomes,

$$\left[-\xi_o C''_*(z) - C_*(z) + \frac{3}{2}\,C_bD\,e^{\frac{-1}{\xi_o}}\right]_{z\,=\,0} = 0 \qquad (3.39)$$

The bed boundary condition for suspended sediment becomes,

$$\left[-\xi_o C'_*(z) + \frac{C_bD}{\xi_o} + \frac{3}{2}\,C_bD\right]_{z\,=\,-1} = 0 \qquad (3.40)$$

and the equation for conservation of bed sediment takes the form,

$$\left[-\xi_o C'_*(z) - C_*(z) + \frac{3}{2}\,C_bD - D\alpha\right]_{z\,=\,-1} = 0 \qquad (3.41)$$

Here, the superscript ' refers differentiation with respect to z.

3.2.9 The Analytical Solution

Equation 3.38 is a second order ordinary differential equation; the solution that satisfies the two boundary conditions, given as equations 3.39 and 3.40, can be found by standard techniques to be,

$$C_*(z) = A_1 e^{s_1 z} + A_2 e^{s_2 z} + S e^{-\left(\frac{1+z}{\xi_o}\right)} \qquad (3.42)$$

where,

$$A_1 = \frac{\left\{C_b D/\xi_o\right\}\left\{\dfrac{(3\xi_o R^2+2R^2+9\xi_o-2)}{(R+1)}\right\} - \left\{3C_b D(R+1)\, e^{\frac{R-1}{2\xi_o}}\right\}}{\left\{(R-1)^2\, e^{-\left(\frac{R-1}{2\xi_o}\right)}\right\} - \left\{(R+1)^2\, e^{\left(\frac{R+1}{2\xi_o}\right)}\right\}} \qquad (3.43)$$

$$A_2 = \frac{\left\{\dfrac{C_b D}{\xi_o}\right\}\left\{\dfrac{(3\xi_o R^2+2R^2+9\xi_o-1)}{(R-1)}\right\} - \left\{3C_b D(R-1)\, e^{-\left(\frac{R+1}{2\xi_o}\right)}\right\}}{\left\{(R-1)^2\, e^{-\left(\frac{R-1}{2\xi_o}\right)}\right\} - \left\{(R+1)^2\, e^{\left(\frac{R+1}{2\xi_o}\right)}\right\}} \qquad (3.44)$$

$$S_1 = \frac{(R-1)}{2\xi_o} \qquad (3.45)$$

$$S_2 = -\frac{(R+1)}{2\xi_o} \qquad (3.46)$$

$$\text{and, } S = \frac{\left(\frac{3}{2}\, C_b D\right)}{\xi_o(\alpha+B\xi_o K^2)} \qquad (3.47)$$

R as defined in equations 3.43, 3.44, 3.45 and 3.46 is given by,

$$R = \sqrt{1 + 4\xi_o(\alpha+B\xi_o k^2)}$$

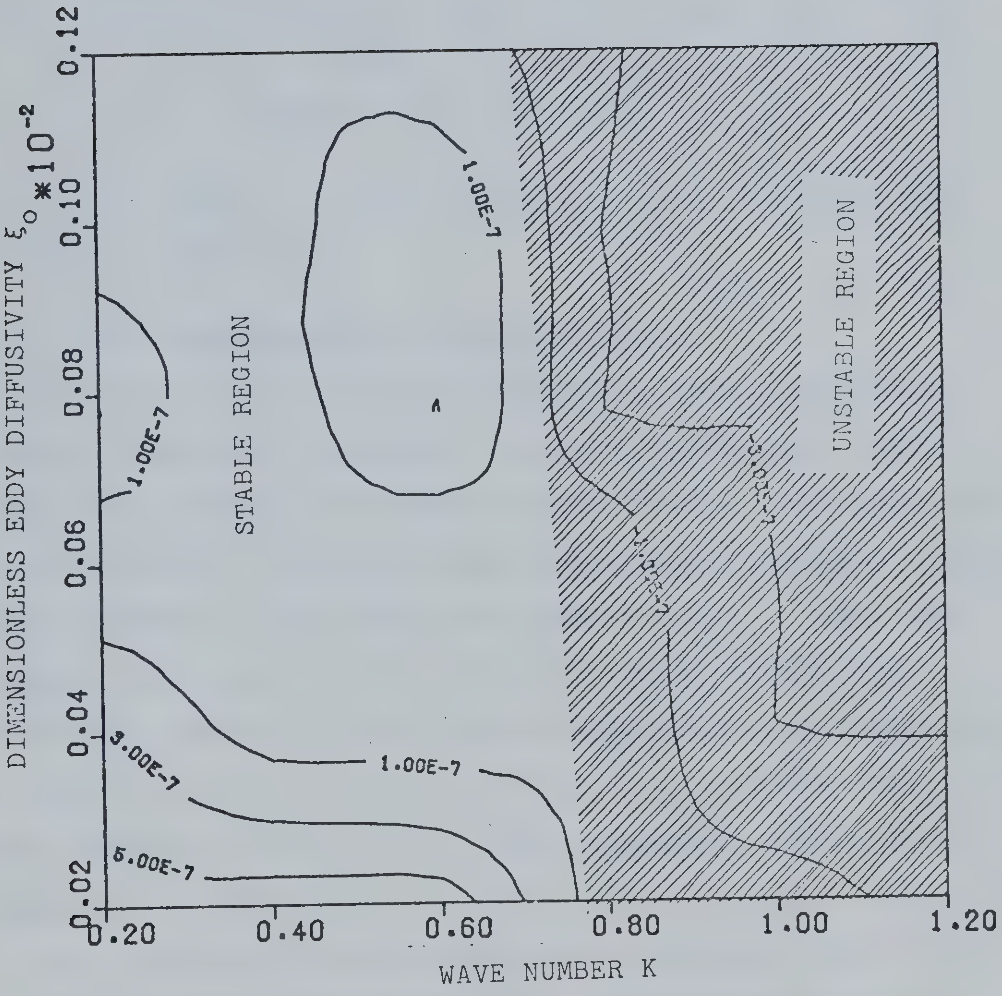

Figure 3.3 Stability Diagram

A solution for α can be obtained by substituting the solution to the main equation (3.42) into the equation of conservation of bed sediment (3.41). Upon reduction, it is found that,

$$\alpha = C_b \left[\frac{\frac{3}{2} - \left\{ \frac{R^2(3\xi_o+2) + (9\xi_o-2)}{2\xi_o} \right\} \left\{ e^{R/\xi_o} - 1 \right\} + 6R\, e^{\frac{R-1}{2\xi_o}}}{(R+1)^2\, e^{R/\xi_o} - (R-1)^2} \right]$$

It is seen that α is a function of ξ_o and k. A contour plot is given in Figure 3.3.

3.2.10 Establishment of Instability

The induced instability of bankforms can be studied in terms of Figure 3.3. Negative values of α indicate stability and positive values of α indicate bank-forming instability in the stream-bed. The figure indicates that at high wave numbers, greater than 0.8, the result is a stable bed. At small wave numbers, less than 0.8, all values of dimensionless eddy diffusivity result in instability giving rise to the deposition of suspended sediment along the sides of the channel and deepening the channel centre. This process is indicative of incipient bank formation.

The result that bank formation is initiated only at small wave number(s) k being given by $\frac{2\pi D_o}{L}$ indicate that channels with high width-depth ratios tend to build their banks.

CHAPTER 4

SUMMARY AND CONCLUSION

An analysis of the process of lateral erosion and deposition of a bed of loose sediment due to flowing water has been presented to explain the geophysical process of bank formation in straight alluvial channels.

The analysis is based upon a consideration of mass balance for suspended sediment. The model assumes the absence of secondary currents affecting the deposition-erosion process. The mass balance equation for suspended sediment is solved subject to appropriate boundary conditions at the water surface and bed. The solution for perturbed suspended sediment concentration is applied to the equation of conservation of bed sediment to evaluate the rate of growth of the amplitude of a sinusoidal bank-like disturbance imposed on the bed. A region of amplitude growth, or bank-forming instability, has been identified. Although when the amplitude becomes finite, the linearized stability theory no longer holds, no attempt has been made to include non-linear effects.

Some results of this study are noted below:

1. Banks can form in rectangular channels as a result of an instability phenomenon. A small bank-like disturbance on an otherwise initially flat bed can initiate deposition on the sides and scour in the centre,

causing initial disturbance to increase in amplitude. The increased amplitude of the disturbance enhances the rate of differential scour and deposition; presumably this process continues, until the banks and channel attain some stable state characteristic of some natural rivers.

2. An interesting aspect of the results of the analysis can explain the lack of bankforms in laboratory flumes. The results indicate that only at low depth-width ratios does bank formation result. The typical depth-width ratios of flumes being high, instability is not expected in laboratory flumes and hence no bankforms are initiated.

REFERENCES

Anderson, A. G. (1953): The Characteristics of Sediment Waves formed by Flow in Open Channels, Proc. Third Midwestern Conference. Fluid Mech., pp. 379-395.

Apman, R. P., and R. R. Rumer, Jr. (1970): Diffusion of Sediment in Developing Flow, Proc. of the American Soc. of Civil Engrs., HY1, pp. 109-123.

Bagnold, R. A., (1955): Some Flume Experiments on Large Grains but Little Denser than the Transporting Fluid, and Their Implications, Proceedings Paper No. 6041, Institute of Civ. Engrs., London, pp. 184.

Beverage, J. P. and J. K. Bulberlson, (1964): Hyperconcentration of Suspended Sediment, Proceedings of the American Society of Civil Engineers, HY6.

Callander, R. A. (1969): Instability and River Channels, J. Fluid Mech. Vol. 36, Part 3, pp. 465-480.

Einstein, H. A., A. G. Anderson, and J. N. Johnson, (1940): A Distinction between Bedload and Suspended load in Natural Streams, Transactions, American Geophysical Union, Vol.

Einstein, H. A., and N. Chien (1954): Second Approximation to the Solution of the Suspended Load Theory, Univ. Calif. Inst. Eng. Res., No. 3.

Engelund, F. A., and E. Hansen (1966): Investigations of Flow in Alluvial Streams, Acta Polytech. Scand., Ci-35.

Engelund, F. A., and E. Hansen (1967): A Monograph on Sediment Transport in Alluvial Streams, Teknisk forlag, Copenhagen.

Engelund, F. (1970): Instability of Erodible Beds, J. Fluid Mech. Vol. 42, part 2, pp. 225-244.

Engelund, F. A., and J. Fredsoe (1971): Three dimensional Stability Analysis of Open-Channel Flow over an Erodible Bed, Nordic Hydrol No. 2, pp. 93-108.

Engelund, F. A. and J. Fredsoe (1974): Transition from Dunes to Plane-bed in Alluvial Channels, Series Paper 4, Inst. Hydrodynamics and Hydraulic Eng., Tech. Univ. Denmark.

Fischer, H. B. (1973): Longitudinal Dispersion and Turbulent Mixing in Open-channel Flow, Annual Review of Fluid Mechanics, No. 8036, pp. 59-77.

Fleming, G. (1970): Particle Size of River Sediments, Proceedings of the American Society of Civil Engineers, HY2.

Gradowczyk, M. H. (1968): Wave Propagation and Boundry Instability in Erodible Bed Channels, J. Fluid Mech. 33, pp. 93-112.

Graf, W. H. (1971): Hydraulics of Sediment Transport, McGraw Hill Book Company.

Guy, H. P., D. B. Simons, and E. V. Richardson (1966): Summary of Alluvial Channel Data from Flume Experiments, 1956-61, Geological Survey Professional Paper 462-I.

Hino, M. (1963): Turbulent Flow with Suspended particles, Proc. Am. Soc. Civil Engrs., Vol. 89, No. HY4.

Hubbell, D. W., and D. Q. Matejka (1959): Investigations of Sediment Transportation, U.S. Geol. Survey, Water Supply Paper, 1476.

Kalinske, A. A., and C. Pien (1943): Experiments on Eddy-Diffusion and Suspended Material Transportation in Open Channels, Trans. Am. Geophys, Union, Vol. 28, No. 4.

Kennedy, J. F. (1963): The Mechanics of Dunes and Antidunes in Erodible-Bed Channels, J. Fluid Mech., No. 16, pp. 521-544.

Kennedy, R. G. (1895): The Prevention of Silting in Irrigation Canals, Min. Proc., Inst. of Civil Engrs., Vol. CXIX.

Lang, H. (1945): 'Hydrodynamics' (1st, Edn., 1879), Dover, New York.

Lane, E. W. (1941): Engineering Calculations of Suspended Sediment, Trans. Am. Geophys. Union, Vol. 20.

Lane, E. W. (1953): Progress Report on Studies on the Design of Stable Channels of the Bureau of Reclamation, Proc. Am. Soc. Civil Engineers, Vol. 79.

Lane, E. W. (1955): Design of Stable Channels, Transactions, ASCE, Vol. 120, Paper No. 2776, pp. 1234-1260.

Nordin, C. F., Jr., (1963): Sediment Transport in Alluvial Channels, A Preliminary Study of Sediment Transport Parameters, Rio Puerco Near Bernardo, New Mexico, Professional Paper 462-C, Geo. Survey, U.S. Dept. of the Interior, Washington, D.C.

Parker, G. N. (1976): On the Cause and Characteristic Scales of Meandering and Braiding in Rivers, J. Fluid Mech., Vol. 76, Part 3, pp. 457-480.

Reynolds, A. J. (1965): Waves on the Erodible Bed of an Open-Channel, J. of Fluid Mech., Vol. 22, Part 1, pp. 113-133.

Raudkivi, A. J. (1966): Bed Forms in Alluvial Channels, J. of Fluid Mech. Vol. 26, Part 3, pp. 507-514.

Reynolds, A. J. (1976): A Decade's Investigation of the Stability of Erodible Stream Beds, Nordic Hydrology, No. 7, pp. 161-180.

Rouse, H. (1964): Sediment Transportation Mechanics: Suspension of Sediment (Progress Report of Task Committee): A Discussion, Proc. Am. Soc. Civil Engrs., Vol. 90, No. HY1.

Sedimentation Engineering (1975): ASCE-Manuals and Reports on Engineering Practice - No. 54.

Shirasuna, T. (1973): Formation of Sand Waves, Proc. 15th Congress Int. Assn. for Hydraulic Research 1, Paper A15, pp. 107-114.

Simons, D. B., E. V. Richardson, and W. L. Laughild, (1963): Studies of Flow in Alluvial Channels, Some Effects of Fine Sediment on Flow Phenomena; Water Supply Paper 498, Geo. Survey, U. S. Dept. of the Interior, Washington, D.C.

Vanoni, V. A. (1975): Sedimentation Engineering, ASCE Report No. 54.

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